Code No. : 041

PRE-BOARD EXAMINATION (JANUARY – 2019)

CLASS: XII **MATHEMATICS** **Time: 3 hrs.**

MAX. MARKS: 100

General Instructions:

*(i) **All** questions are compulsory.*
*(ii) This question paper contains **29** questions.*
*(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.*
*(iv) Questions **5-12** in **Section B** are short-answer type questions carrying **2** marks each.*
*(v) Questions **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.*
*(vi)Questions **24-29** in **Section D are** long-answer-**II** type questions carrying **6** marks each.*

SECTION A

Question numbers 1 to 4 carry 1 mark each.

1. If $a * b$ denotes the larger of 'a' and 'b' and if $aob = (a* b) + 3$, then write the value of $(5)\ o(10)$, where $*$ and o are binary operations. (1)

2. If $\begin{bmatrix} x+3y & y \\ 7-x & 5 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & 5 \end{bmatrix}$, find the value of y – x (1)

3. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ (1)

4. Determine the direction cosines of the line which make equal angles with the positive direction of the co – ordinate axis. (1)

OR

Find the acute angle which the line with direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6},}\ n$ makes with positive direction of z-axis.

SECTION B

Question numbers 5 to 12 carry 2 marks each.

5. Using properties of determinants, prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$ (2)

6. Prove that : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3),\ x \in [\frac{-1}{2}, \frac{1}{2}]$ (2)

OR

Differentiate $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x.

7. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0{\cdot}005x^3 - 0{\cdot}02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output. (2)

8. If $\sin^{-1} x + \cos^{-1} x = \pi$, then find the value of x. (2)

9. Find the projection (vector) of $2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\hat{\imath} - 2\hat{\jmath} + \hat{k}$. (2)

OR

Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

10. Find $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos x}\, dx$ (2)

11. Find the value of 'a', if the function $f(x)$ defined by (2)

$$f(x) = \begin{Bmatrix} 2x-1, x<2 \\ a, \quad x=2 \\ x+1, x>2 \end{Bmatrix} \textit{ is continuous at } x = 2$$

12. Evaluate $\int \frac{2\cos x}{3sin^2 x}\ dx$ (2)

OR

Find : $\int \frac{e^x(x-3)}{(x-1)^3}\ dx$

SECTION C

Question numbers 13 to 23 carry 4 marks each.

13. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$ (4)

Then find the value of $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}$?

14. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III. (4)

15. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$,then prove that (4)

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$, and hence show that $[\vec{a} \quad \vec{b} \quad \vec{c}]=0$.

16. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1,1,1). (4)

17. If $y=\log(\sqrt{x}+\frac{1}{\sqrt{x}})^2$, then prove that $x(x+1)^2y_2+\ (x+1)^2y_1=2$. (4)

18. If the function $f: R\to R$ is given by $f(x)=x^2+3x+1$ and $g: R\to R$ is given by $g(x)=2x-3$, find (i)fog and $(ii)gof$. (4)

19. If $(cosx)^y=(\cos y)^x$,find $\frac{dy}{dx}$ (4)

OR

If $\sin y=x\sin(a+y)$,prove that $\frac{dy}{dx}=\frac{sin^2(a+y)}{\sin a}$

20. Find the point on the curve $y=x^3-11x+5$ at which the equation of tangent is (4)

$y=x-11$.

OR

Using differentials, find the approximate value of $\sqrt{49.5}$

21. Evaluate : $\int\frac{2}{(1-x)(1+x^2)}dx$ (4)

22. Solve the following differential equation: (4)

$x\frac{dy}{dx}+y=x\log x, x\neq 0.$

23. The length x of a rectangle is decreasing at the rate of 5cm/minute and the width y is increasing at the rate of 4cm/minute. When $x = 8$cm, find the rate of change of (a) the perimeter (b) area of the rectangle. (4)

OR

Find the intervals in which the function f given by $f(x) = \sin x + \cos x$,

$0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.

SECTION D

Question numbers 24 to 29 carry 6 marks each.

24. Using matrices, solve the following system of linear equations : (6)

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

OR

Using elementary operations, find the inverse of the following matrix :

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

25. Evaluate : (6)

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9\sin 2x}\, dx$$

OR

Evaluate:

$$\int_1^3 (x^2 + 3x + e^x).\, dx$$

26. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an 'A' grade, what is the probability that the student is a hostler? (6)

27. Using integration, find the area in the first quadrant bounded by the curve $y = x|x|$, the circle $x^2 + y^2 = 2$ and the $y - axis$. (6)

OR

Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 8,\ x^2 \leq 2y\}$

28. Find the distance of point $-2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ from the line (6)

$\vec{r} = \hat{\imath} + 2\hat{\jmath} - \hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - 9\hat{k})$ measured parallel to the plane

$x - y + 2z - 3 = 0.$

29. A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit, 1/8 of an hour in quality control work and Rs1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and ₹ 0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of ₹ 9 per unit and the second product described has a market value (sale price) of ₹ 8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit. (6)